**Statistics**

**Lecture 16**

Feb 19-8:47 AM

SG 16

**Binomial Prob. dist.**

1) we have $n$ independents evens (Trials)

2) Each event has only two outcomes.

$P(\text{Success}) = p$ $\quad$ $P(\text{Failure}) = q$

3) $p+q=1$, $q=1-p$, $p \& q$ remain unchanged for all $n$ events

4) $x \rightarrow$ # of Successes, $n-x \rightarrow$ # of failures

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

Consider a binomial Prob. dist with $n=10$, $p=.6$

$P(x=7) = {}_{10}C_7 \cdot (.6)^7 \cdot (.4)^3 = .215$

($n$, $x$, $p$, $q$, $x$, $n-x$)

10 MATH PRB $_nC_r$ 7 $\times$ .6 $\wedge$ 7 $\times$ .4 $\wedge$ 3 $\rightarrow$ Enter

Compute

$np = 10(.6) = 6$

$npq = 10(.6)(.4) = 2.4$

$\sqrt{npq} = \sqrt{2.4} \approx 1.549$

Nov 16-7:22 AM

Consider a binomial prob. dist with $n=20$ and $P=.5$

1) $q = 1-P$
$= 1-.5 = \boxed{.5}$

2) $np = 20(.5)$
$= \boxed{10}$

3) $npq = 20(.5)(.5)$
$= \boxed{5}$

4) $\sqrt{npq} = \sqrt{5} \approx \boxed{2.236}$

5) $P(x=12) = {}_{20}C_{12} \cdot (.5)^{12} \cdot (.5)^{8} = \boxed{.120}$

($n$, $x$, $P$, $q$; $x$, $n-x$)

Now using TI Command

2nd [VARS] ↓ binomPdf( 20, .5, 12) Enter

Trials: 20
P: .5
x-value: 12
Paste

Your work

$P(x=12) = \text{binompdf}(20, .5, 12) = .120$

Nov 16-7:35 AM

Consider a binomial Prob. dist. with $n=75$ and $P=.6$.

1) $q = 1-P$
$= 1-.6 = \boxed{.4}$

2) $np = 75(.6)$
$= \boxed{45}$

3) $npq = 75(.6)(.4)$
$= \boxed{18}$

4) $\sqrt{npq} = \sqrt{18} \approx \boxed{4.243}$

5) $P(x=50) = \text{binompdf}(75, .6, 50) = \boxed{.048}$

($n$, $P$, $x$)

6) $P(x \le 50) = P(x=50) + P(x=49) + P(x=48) + \cdots + P(x=0)$
$= \text{binomcdf}(75, .6, 50) = \boxed{.904}$

At most

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Consider a binomial Prob. dist. with $n=100$ & $P=.5$

1) $q = 1-P$ $= .5$

2) $np = 100(.5)$ $= 50$

3) $npq = 100(.5)(.5)$ $= 25$

4) $\sqrt{npq} = \sqrt{25} = 5$

5) $P(x=60) = \text{binompdf}(100, .5, 60) = .011$

6) $P(x \leq 60) = P(x \leq 59) = \text{binomcdf}(100, .5, 59)$

(60 below; 59 at most)

$= .972$

7) $P(x \geq 45) = 1 - P(x \leq 44)$

(45 at least; 1 Total Prob.)

$= 1 - \text{binomcdf}(100, .5, 44)$

$= .864$

We don't want | We want (45 … 44)

Nov 16-7:57 AM

You are taking a multiple-choice exam with 40 questions.

Each question has 4 choices but only one correct choice and you are making random guesses.

1) $n = 40$

2) $p = \frac{1}{4} = .25$

3) $q = \frac{3}{4} = .75$

4) $np = 40 \cdot (.25)$ $= 10$

5) $npq = 40(.25)(.75)$ $= 7.5$

6) $\sqrt{npq} = \sqrt{7.5}$ $\approx 2.739$

7) P(guess exactly 15 correct ans.)

$= P(x = 15) = \text{binompdf}(40, .25, 15) = .028$

8) P(guess at most 20 correct ans.)

$= P(x \leq 20) = \text{binomcdf}(40, .25, 20) = .9998$

$\approx 1$

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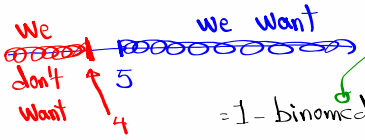
9) P(at least 5 Correct ans.)

= P( $x \geq 5$) = 1 − P($x \leq 4$)  (Total Prob.)




= 1 − binomcdf(40, .25, 4) = .984

10) P(guess more than 8 Correct ans.)

= P( $x > 8$) = P($x \geq 9$) = 1 − P($x \leq 8$)

at least

= 1 − binomcdf(40, .25, 8)


= .700

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You flip a fair coin 400 times. P = .5  q = .5

Success is to land tails.

1) n = 400  2) p = .5  3) q = .5

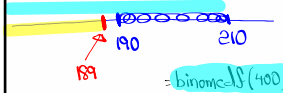
4) np = 400(.5) = 200

5) npq = 400(.5)(.5) = 100

6) $\sqrt{npq} = \sqrt{100} = 10$

5) P(get between 190 and 210 tails, inclusive)

P( $190 \leq x \leq 210$) = P($x \leq 210$) − P($x \leq 189$)

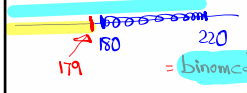



= binomcdf(400, .5, 210) − binomcdf(400, .5, 189)

= .706

6) P(get between 180 and 220 tails, inclusive)

= P( $180 \leq x \leq 220$) = P($x \leq 220$) − P($x \leq 179$)




= binomcdf(400, .5, 220) − binomcdf(400, .5, 179)

= .960

Nov 16-8:26 AM

Binomial Prob. dist.:

Mean $\mu$

Variance $\sigma^2$

Standard deviation $\sigma$

$\mu = np$

$\sigma^2 = npq$

$\sigma = \sqrt{\sigma^2}$

Consider a binomial Prob. dist. with $n=150$, $P=.8$

$q = 1 - P = 1 - .8 = .2$ $\quad \mu = np = 150(.8) = 120$

$\sigma^2 = npq = 150(.8)(.2) = 24$ $\quad \sigma = \sqrt{\sigma^2} = \sqrt{24} \approx 5$

By Empirical Rule

68% Range $= \mu \pm \sigma = 120 \pm 5 \rightarrow$ 115 To 125

95% Range $= \mu \pm 2\sigma = 120 \pm 2(5) \rightarrow$ 110 To 130 (Usual Range)

Nov 16-8:39 AM

You are taking a True-False exam with 100 questions. $P = .5$ $q = .5$

You are making random guesses.

1) $n = 100$ 2) $P = .5$ 3) $q = .5$

4) $\mu = np = 100(.5) = 50$

5) $\sigma^2 = npq = 100(.5)(.5) = 25$

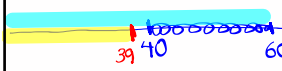
6) $\sigma = \sqrt{\sigma^2} = \sqrt{25} = 5$

7) Find the Usual Range (95% Range)

$\mu \pm 2\sigma = 50 \pm 2(5) = 50 \pm 10 \rightarrow$ 40 To 60

8) Find P(Correctly guess between 40 and 60 ans, inclusive)

$P(40 \le x \le 60) = P(x \le 60) - P(x \le 39)$




$= \text{binomcdf}(100, .5, 60) - \text{binomcdf}(100, .5, 39)$

$= .965 = 96.5\%$

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Consider a binomial Prob. dist. with $n=250$ & $p=.6$

1) $q = .4$

2) $\mu = 250(.6) = 150$

3) $\sigma^2 = 250(.6)(.4) = 60$

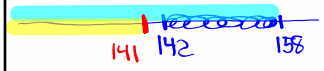
4) $\sigma = \sqrt{\sigma^2} = \sqrt{60} \approx 7.746$

Round up $\mu$ & $\sigma$ to a whole #

$\mu = 150$ , $\sigma = 8$

68% Range = $\mu \pm \sigma = 150 \pm 8 \rightarrow$ 68% Range: 142 to 158

$P(142 \le x \le 158) = P(x \le 158) - P(x \le 141)$




= binomcdf(250, .6, 158) − binomcdf(250, .6, 141)

= .728 ≈ 73%

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